**КАЗАХСКИЙ НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ ИМ. АЛЬ-ФАРАБИ**

**Механико-математический факультет**

**Кафедра дифференциальных уравнений и теории управления**

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|  | Утверждаю Декан факультета:  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Жакебаев Д.Б.  «\_\_\_\_»\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2018 |

# УЧЕБНО-МЕТОДИЧЕСКИЙ КОМПЛЕКС ДИСЦИПЛИНЫ

### VIMO4414 «Вариационное исчисление и методы оптимизации»

**Специальность: «5B060100 – Математика»**

**Образовательная программа: Проектирование и разработка ПО**

**Курс – 4**

**Семестр – 7**

**Кол-во кредитов – 3**

**Форма обучения дневная**

**Алматы 2018 г.**

Учебно-методический комплекс дисциплины составил д.ф.м.н профессор кафедры Серовайский С.Я.

На основании рабочего учебного плана по специальности «5B060100 – Математика».

Рассмотрен и рекомендован на заседании кафедры Дифференциальных уравнений и теории управления

от «\_\_\_ » \_\_\_\_\_\_\_\_\_\_\_\_\_\_ 2018 г., протокол №\_\_\_

Заведующий кафедрой \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Х. Хомпыш

### Рекомендовано методическим бюро факультета

«\_\_\_\_» \_\_\_\_\_\_\_\_\_\_\_ 2018 г., протокол №\_\_\_

Председатель методбюро факультета \_\_\_\_\_\_\_\_\_\_­­­­­ Кушербаева У.Р.

**AL-FARABI KAZAKH NATIONAL UNIVERSITY**

**Faculty of Mechanics and Mathematics**

**Department of Fundamental Mathematics**

**Educational program in the specialty «5B060100-Mathematics».**

**SYLLABUS**

**Calculus of variations and optimization methods**

**Fall semester, 2018-2019**

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| **Discipline’s code** | **Discipline’s title** | **Type** | **No. of hours per week** | | | | **Number of credits** | | ECTS |
| **Lect.** | **Pract.** | | **Lab.** |
|  | Calculus of variations and optimization methods |  | 2 | 1 | |  | 3 | |  |
| **Prerequisites** | Mathematical analysis, functional analysis, differential equations, numerical methods | | | | | | | | |
| **Lecturer** | S. Serovajsky | | | | **Office hours** | | | Scheduled | |
| **e-mail** | [serovajskys@mail.ru](mailto:serovajskys@mail.ru) | | | |
| **phone** | +7 701 8315197 | | | | **Auditory** | | | room 4 | |

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| **Description of the discipline** | Analysis of general methods of calculus of variations and optimization control theory |
| **Course Objective** | The main purpose of the course is to familiarize students with the fundamental concepts of calculus of variations and optimization control theory |
| **Learning Outcomes** | By the end the course, students should be able to:   * To know the applications of the extremum theory; * To know the classification of the problems of the extremum theory; * To be able to analyze the extremum of functions; * To use the variational method for solving problems of minimization of integral functionals; * To know the problem statements of optimal control problems; * To know general optimization methods; * To know approximation methods for optimal control problems. |

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| **Literature and Information resources** | 1. Алексеев В. М., Тихомиров В. М., Фомин С. В. Оптимальное управление. – М., Наука, 1979. 2. Будылин А.М. Вариационное исчисление. – Санкт-Петербург, СПбГУ, 2001. 3. Васильев Ф.П. Методы оптимизации. В двух томах. – М.: МЦНМО, 2011. 4. Лутманов С.В. Курс лекций по методам оптимизации. – Ижевск, 2001. 5. Эльсгольц Л.Э. Дифференциальные уравнения и вариационное исчисление. – М., Наука, 1969. 6. Kirk D. E. Optimal Control Theory: An Introduction. – New Jersey, Englewood Cliffs, 2004. 7. Serovajsky S. Practical Course of the Optimal Control Theory with Examples. Almaty, Қазақ университеті, 2011. 8. [http://www.newlibrary.ru/book/budylin\_a\_m\_/variacionnoe\_ischislenie.html](http://www.newlibrary.ru/book/budylin_a_m_/variacionnoe_ischislenie.html%20) . | |
| **Organization of the course** | This course is an introductory course, where a general acquaintance with a large volume of theoretical and practical material is given. In the preparation for the discipline, an essential role is given to the textbook and the collection of problems. Sufficient attention is also paid to the actual solution of problems.  Two sets of homework assignments (in the form of a set of tasks) will give you the opportunity to fully in-depth acquaintance with the practical application of theoretical material. | |
| **Course Requirements** | You must present your assignments in written form before the deadline announced by teacher. The mark is given only after passing of the SIS in a form of quiz. Homework should be done in a thin notebook. Problems with solutions must be numbered and ordered. It is important that you show the work in an organized manner clearly showing the final answer with appropriate units. Final answers should be highlighted. Students may collaborate solving homework on the condition that each student actively works on solving of each problem and is able to give clear explanation for the solution of any problem.  For consultations on the implementation of homework, as well as additional information on the studied material, and all other questions, please contact the course instructor during his office hours.  Students with disabilities may receive advice on e-mail: [serovajskys@mail.ru](mailto:serovajskys@mail.ru) |
| **Evaluation system** | Criteria-based evaluation: assessment of learning outcomes in correlation with descriptors (verification of formation of competences on attestation controls and examinations).  Summative assessment: evaluation of attendance and activity in the classroom; evaluation of assignments and Student’s Individual Studies (SIS1, SIS2).  These types of evaluation are given in the table below:   |  |  | | --- | --- | | Types of work | % | | Attendance | 11% | | Active participation in the class work | 10% | | Homework (SIS-1, SIS-2) | 9% | | Control works (Quiz-1,2; Test-1,2) | 30% | | Exams | 40% | | TOTAL | 100% |   Your final grade is calculated by the formula:  Total = 0.6\*(At1+At2)\2+0.1\*MidTermExam+0.3\*FinalExam    The final grade will be calculated according to the evaluation system accepted in University:  95% - 100%: А 90% - 94%: А-  85% - 89%: В+ 80% - 84%: В 75% - 79%: В-  70% - 74%: С+ 65% - 69%: С 60% - 64%: С-  55% - 59%: D+ 50% - 54%: D- 0% -49%: F |
| **Policy of Discipline** | Cellular phones must be silenced during lecture or seminar. Regular and punctual attendance at all scheduled classes is expected. Attendance will be taken regularly. Students should consult with the instructor when an unavoidable absence due to an emergency or illness occurs. Deadlines of homework or control works can be prolonged in the case of circumstances such as illness, emergency, unforeseen events, etc. in accordance with the University's academic policy.  In order to maintain an excellent working environment, students are expected to be respectful and courteous to each other. Formulate your objections in correct manner. Plagiarism and other forms of cheating are not allowed. Any cheating is unacceptable during tests, quizzes and exams. Student convicted of falsifying any information of the course will receive a final grade «F». |

**STRUCTURE AND CONTENT OF DISCIPLINE**

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| **Week** | **Topics** | **Hours** | **Max point** |
| 1 | **Lecture 1. Practical examples of the extremum problems**. Maximization of the flight of the body. Brachistochrone problem. Maximization of the flight of the missile. | 2 | 0 |
| **Practical work 1**. Practical examples of the extremum problems. | 1 | 3 |
| **Homework 1**. Practical examples of the extremum problems. |  | 10 |
| 2 | **Lecture 2. Minimization of functions**. Stationary condition. Examples. Maximization of the flight of the body. Minimization of the function of many variables. | 2 | 1 |
| **Practical work 2**. Minimization of functions and stationary condition. | 1 | 3 |
| **Homework 2**. Use stationary condition for the concrete function*.* Check the properties of the stationary points. Chose the function with given property. |  | 10 |
| 3 | **Lecture 3. Euler equation for Lagrange problem.** Lagrange problem. Euler equation. Examples. The fall of the body. Fermat principle and the refraction of light law. | 2 | 2 |
| **Practical work 3**. Euler equation for Lagrange problem. | 1 | 3 |
| **Homework 3.** Determine Euler equation for the concrete Lagrange problem. Find the general solution of Euler equation, which depends from two constants. Find these constants by means of the given boundary conditions. Find the corresponding solution of the boundary problem. Calculate the corresponding value of the given functional. Calculate the value of the given functional for the linear function which satisfies the given boundary conditions. Compare these results. |  | 10 |

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| 4 | **Lecture 4. Lagrange problem for the functions family.** Problem statement. The system of Euler equations. Example. Principle of the least action. | 2 | 1 |
|  | **Practical work 4**. Lagrange problem for the functions family. | 1 | 3 |
|  | **Homework 4**. Determine the system of Euler equations for the concrete problem. Find general solution of this system. Find the solution of Euler equations, which satisfies boundary conditions. Show the graphs of these solutions. Calculate the corresponding value of the given integral. |  | 10 |
| 5 | **Lecture 5. Lagrange problem with high derivatives.** Problem statement. Euler – Poisson Equation. Example. Bending of the elastic beam. | 2 | 2 |
|  | **Practical work 5**. Lagrange problem with high derivatives. | 1 | 3 |
|  | **Homework 5**. Determine the system of Euler – Poisson equation for the concrete problem. Find general solution of this equation. Find the solution of Euler – Poisson equation, which satisfies given boundary conditions. Show the graph of this solution. Calculate the corresponding value of the given integral. |  | 10 |
| 6 | **Lecture 6. Lagrange Problem for functions with many variables.** Problem statement. Ostrogradsky equation. Dirichlet integral. The oscillation of the string. | 2 | 1 |
|  | **Practical work 6.** Lagrange Problem for functions with many variables. | 1 | 3 |
|  | **Homework 6.** Determine Ostrogradsky equation for the concrete problem. |  | 10 |
| 7 | **Lecture 7. Bolza Problem.** Problem statement. Necessary conditions of extremum. Transversality conditions. Example. River crossing problem. | 2 | 2 |
|  | **Practical work 7.** Bolza Problem. | 1 | 3 |
|  | **Homework 7.** Determine Euler equation and the transversality conditions for the concrete problem. Find the general solution of this equation. Find the solution of boundary problem. Calculate the corresponding value of the given integral. |  | 10 |
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|  | Border control 1 |  | 100 |
| 8 | **Lecture 8. Variational problems with isoperimetric conditions.** Problems with isoperimetric condition. Lagrange multipliers method. A spectrum problem. The problem with many isoperimetric conditions. | 2 | 1 |
|  | **Practical work 8.** Variational problems with isoperimetric conditions. | 1 | 3 |
|  | **Homework 8.** Determine Euler equation for the concrete problem. Verify the sign of the Lagrange multiplier with using multiplication of Euler equation by unknown function and integration. Find the general solution of Euler equation; it depends from two constants and Lagrange multiplier. Using given boundary conditions and isoperimetric condition find three unknown constants. Find the set of the solutions of the conditions of the extremum. Calculate the value of the given integral for all solution of the conditions of the extremum. |  | 8 |
| 9 | **Lecture 9. Variational problems with pointwise constraints.** Problem statement. Lagrange multipliers method. Example. Oscillation of the pendulum. | 2 | 2 |
|  | **Practical work 9.** Variational problems with pointwise constraints. | 1 | 3 |
|  | **Homework 9.** Denote the system of the extremum conditions (concrete Euler equations with boundary and addition conditions). Multiply the first Euler equation by the given parameter a, and second equation by b. Add these equalities with using of the condition (\*). Find Lagrange multiplier λ. Put λ to Euler equations. Find the general solutions of two Euler equations. It equals the sum of the general solution of the corresponding homogeneous equation and the constant, which satisfies the given equation. Find four constants from general solutions of Euler equations with using of the boundary conditions. Put these constants to the formulas of the general solutions. It will be the solution of the problem. | 0 | 8 |
| 10 | **Lecture 10. Easiest optimization control problems.** Maximization of the flight of the missile (problem statement). Pontyagin’s maximum principle. Example. Iterative method for solving the optimality conditions. | 2 | 1 |
|  | **Practical work 10.** Easiest optimization control problems. | 1 | 3 |
|  | **Homework 10.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality. |  | 8 |
| 11 | **Lecture 11. Optimization control problems for the vector case.** Problem statement. Pontyagin’s maximum principle. Example. Maximization of the flight of the missile (solving). | 2 | 2 |
|  | **Practical work 11.** Optimization control problems for the vector case. | 1 | 3 |
|  | **Homework 11.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality. |  | 8 |
| 12 | **Lecture 12. Optimization control problem with fixed final state.** Problem Statement.Maximum principle. Example. Time optimization problem. Firing method. | 2 | 1 |
|  | **Practical work 12.** Optimization control problem with fixed final state. | 1 | 3 |
|  | **Homework 12.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality with using of firing method. |  | 8 |
| 13 | **Lecture 13. Differentiation of functionals and abstract optimization problems.** Gradient methods for functions. Gateau derivatives of functionals. Examples. Gradient methods for functionals. | 2 | 2 |
|  | **Practical work 13.** Differentiation of functionals and abstract optimization problems. | 1 | 3 |
|  | **Homework 13.** Calculate Gateau derivative for the concrete functional. Determine gradient method and projection gradient method. |  | 8 |
| 14 | **Lecture 14**. **Variational inequalities**. Variational inequalities and constraints minimization of functional. Examples. Variational inequalities and constraints minimization of functional. | 2 | 1 |
|  | **Practical work 14.** Variational inequalities. | 1 | 3 |
|  | **Homework 14.** Calculate Gateau derivative for the concrete functional. Determine variational inequality. Find the solution of variational inequality. Calculate the value of minimizing functional. |  | 8 |
| 15 | **Lecture 15. Existence and uniqueness of extremum problems.** Existence theorem for abstract optimization problems. Uniqueness theorem for abstract optimization problems. Example. | 2 | 2 |
|  | **Practical work 15.** Existence and uniqueness of extremum problems. | 1 | 3 |
|  | **Homework 15.** Prove the convexity and continuity of the concrete functional. Prove the convexity, closeness and boundedness of the concrete set of admissible control. Prove the existence of the optimal control. |  | 8 |
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|  | Border control 2 |  | 100 |

Dean of the Faculty

Chairman of the Faculty Methodical Bureau

Head of the Department

Lecturer: